## George Mason University Department of SEOR and Mathematical Sciences Department Spring 2013 Professor Roman A. Polyak

## MATH 493-002/ECON 496-006/SYST 465-001: Pricing in Optimization and Game Theory Friday 1:30-4:10 pm, West 1004

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**Office Hours**: Thursday 3 pm-5 pm or by appointment. E-mail: *rpolyak@gmu.edu* 

**Text**: Wayne Winston, M.Venkataramanan "Introduction to Mathematical Programming", Fourth Edition Book/Cole Thomson Learning Inc. 2003.

**Course Summary:** Finding an adequate mechanism for pricing limited recourses, goods and services is one of the main goals of planning complex systems. On the other hand pricing is the key tool for finding optimal solutions and economic equilibrium. It reflects the fundamental role of Lagrange multipliers in constrained optimization.

In the first half of the course we will cover the basic ideas and methods of Linear Programming (LP) or Linear Equilibrium (LE). The fundamental role of pricing in LP will be particularly emphasized: duality, sensitivity analysis and decomposition.

We will show then the intimate connections between solving the dual pair of LP and finding equilibrium in two persons Matrix Game (MG).

In the second half we will consider the Linear Exchange (LE) model. The role of pricing in establishing economic equilibrium in LE will be emphasized. Application multipliers method for solving correspondent convex optimization problem leads to a pricing mechanism for establishing economic equilibrium in LE.

We conclude the course by introducing Nonlinear Equilibrium (NE) and showing two pricing methods for optimal allocation limited resources using NE.

We will have homework assignment and projects.

Grading: 15% homework; 30% mid-term exam; 20 % project; 35 % final exam.

## **Course Schedule**

- 1. Real life applications that led to LP and NLP formulation
- 2. Simplex method
- 3. Shadow prices, sensitivity analysis (review)
- 4. Duality in LP: basic duality theorems and their economic interpretation
- 5. Pricing mechanism in LP. Dantzig-Wolf decomposition
- 6. Two person MG. Pure and mixed strategies. The basic John Von Newman theorem for MG
- 7. MG and duality in LP. Solving MG using LP methods **Midterm**
- 8. Braun-Robinson iterative method for solving MG.
- 9. Basics in NLP: KKT conditions and saddle point of the Classical Lagrangian.
- 10. Lagrange multipliers method for convex optimization.
- 11. Linear Exchange as a convex optimization problem; Pricing method for finding Equilibrium in LE model.
- 12. Nonlinear Equilibrium for optimal resource allocation. Two pricing methods

for finding NE and their economic interpretation.

## Final Exam: May 10, 2013.