Operations research is concerned with

optimal decision making in and modeling of

deterministic and probabolistic systems that

originate from Real Life.

"Linear programming has been one of the most important postwar developments in economic theory."

Robert Dorfman, Paul Samuelson, Robert Solow

"If one would take statistics about which mathematical problem is using up most of the computer time in the world then (not including database handling problems like sorting and searching) the answer would probably be linear programming."

Laszlo Lovasz, Princeton

"It (linear programming) is used to allocate resources, plan production, schedule workers, plan investment portfolios and formulate marketing (and military) strategies. The versatility and economic impact of linear programming in today's industrial world is truly awesome."

Eugene Lawler, Berkeley

TABLE 1.1 Some applications of operations research

Organization	Nature of Application	Year of Publication*	Related Chapters†	Annual Savings
The Netherlands Rijkswaterstaat	Develop national water management policy, including mix of new facilities, operating procedures, and pricing.	1985	2–8, 13, 22	$15 million
Monsanto Corp.	Optimize production operations in chemical plants to meet production targets with minimum cost.	1985	2, 12	$2 million
United Airlines	Schedule shift work at reservation offices and airports to meet customer needs with minimum cost.	1986	2–9, 12, 17, 18, 20	$6 million
Citgo Petroleum Corp.	Optimize refinery operations and the supply, distribution, and marketing of products.	1987	2–9, 20	$70 million
San Francisco Police Department	Optimally schedule and deploy police patrol officers with a computerized system.	1989	2–4, 12, 20	$11 million
Texaco, Inc.	Optimally blend available ingredients into gasoline products to meet quality and sales requirements.	1989	2, 13	$30 million
IBM	Integrate a national network of spare parts inventories to improve service support.	1990	2, 19, 22	$20 million +$250 million less inventory
Yellow Freight System, Inc.	Optimize the design of a national trucking network and the routing of shipments.	1992	2, 9, 13, 20, 22	$17.3 million
New Haven Health Department	Design an effective needle exchange program to combat the spread of HIV/AIDS.	1993	2	33% less HIV/AIDS
AT&T	Develop a PC-based system to guide business customers in designing their call centers.	1993	17, 18, 22	$750 million
Delta Airlines	Maximize the profit from assigning airplane types to over 2500 domestic flights.	1994	12	$100 million
Digital Equipment Corp.	Restructure the global supply chain of suppliers, plants, distribution centers, potential sites, and market areas.	1995	12	$800 million
China	Optimally select and schedule massive projects for meeting the country's future energy needs.	1995	12	$425 million
South African defense force	Optimally redesign the size and shape of the defense force and its weapons systems.	1997	12	$1.1 billion
Proctor and Gamble	Redesign the North American production and distribution system to reduce costs and improve speed to market.	1997	8	$200 million
Taco Bell	Optimally schedule employees to provide desired customer service at a minimum cost.	1998	12, 20, 22	$13 million
Hewlett-Packard	Redesign the sizes and locations of buffers in a printer production line to meet production goals.	1998	17, 18	$280 million more revenue

*Pertains to a January–February issue of *Interfaces* in which a complete article can be found describing the application.
†Refers to chapters in this book that describe the kinds of OR techniques used in the application.

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Monsanto Corp.	Optimize production operations in chemical plants to meet production targets with minimum cost.	1985	2, 12	$2 million
Weyerhaeuser Co.	Optimize how trees are cut into wood products to maximize their yield.	1986	2, 10	$15 million
Eletrobras/CEPAL, Brazil	Optimally allocate hydro and thermal resources in the national electrical generating system.	1986	10	$43 million
United Airlines	Schedule shift work at reservation offices and airports to meet customer needs with minimum cost.	1986	2–9, 12, 15, 16, 18	$6 million
Citgo Petroleum Corp.	Optimize refinery operations and the supply, distribution, and marketing of products.	1987	2–9, 18	$70 million
SANTOS, Ltd., Australia	Optimize capital investments for producing natural gas over a 25-year period.	1987	2–6, 13, 21	$3 million
San Francisco Police Department	Optimally schedule and deploy police patrol officers with a computerized system.	1989	2–4, 12, 18	$11 million
Electric Power Research Institute	Manage oil and coal inventories for electric utilities to balance inventory costs and risk of shortages.	1989	17, 21	$59 million
Texaco, Inc.	Optimally blend available ingredients into gasoline products to meet quality and sales requirements.	1989	2, 13	$30 million
IBM	Integrate a national network of spare-parts inventories to improve service support.	1990	2, 17, 21	$20 million + $250 million less inventory
Yellow Freight System, Inc.	Optimize the design of a national trucking network and the routing of shipments.	1992	2, 9, 13, 18, 21	$17.3 million
U.S. Military Airlift Command	Quickly coordinate aircraft, crews, cargo, and passengers to run the Operation Desert Storm airlift.	1992	10	Victory
American Airlines	Design a system of fare structures, overbooking, and coordinating flights to increase revenues.	1992	2, 10, 12, 17, 18	$500 million more revenue
New Haven Health Dept.	Design an effective needle exchange program to combat the spread of HIV/AIDS.	1993	2	33% less HIV/AIDS

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Introduction

Linear programming is one of the great success stories of optimization. Since its formulation in the 1930s and 1940s and the development of the simplex algorithm by Dantzig in the mid 1940s, generations of workers in economics, finance, and engineering have been trained to formulate and solve linear programming problems. Even when the situations being modeled are actually nonlinear, linear formulations are favored because the software is highly sophisticated, because the algorithms guarantee convergence to a global minimum, and because uncertainties in the model and data often make it impractical to construct a more elaborate nonlinear model.

The publication in 1984 of Karmarkar's paper [57] was probably the most significant event in linear programming since the discovery of the simplex method. The excitement that surrounded this paper was due partly to a theoretical property of Karmarkar's algorithm—polynomial complexity—and partly to the author's claims of excellent practical performance on large linear programs. These claims were never fully borne out, but the paper sparked a revolution in linear programming research that led to theoretical and computational advances on many fronts. Karmarkar's paper, and earlier work whose importance was recognized belatedly, gave rise to the field of *interior-point methods*, and the years following 1984 saw rapid development and expansion of this new field that continue even today. Theoreticians focused much of their attention on *primal-dual methods*, the elegant and powerful class of interior-point methods that is the subject of this book. Computational experiments, which took place simultaneously with the theoretical development, showed that primal-dual algorithms also performed better than other interior-point methods on practical problems, outperform-

Newton	(1642 - 1727)
Lagrange	(1736 - 1813)
Legendre	(1752 - 1833)
Fourier	(1758 - 1830)
Gauss	(1777 - 1855)
Jordan	(1833 - 1922)

L. V. Kantorovich (1912 - 1986)
George Dantzig (1914 - ...)

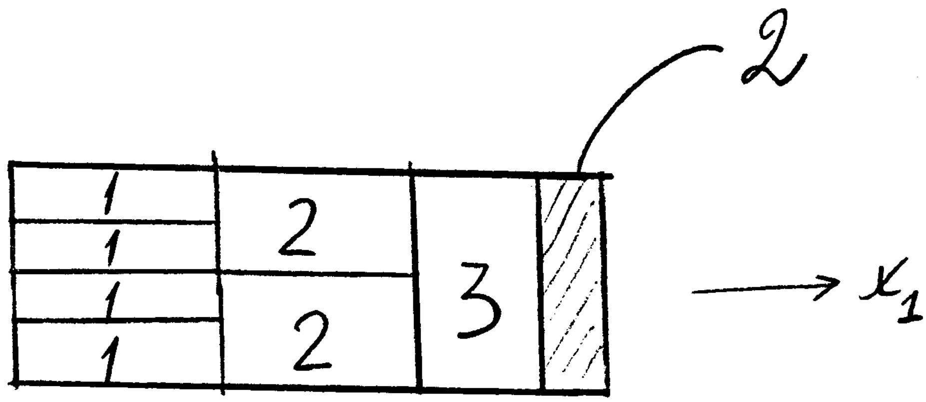
The Pioneer

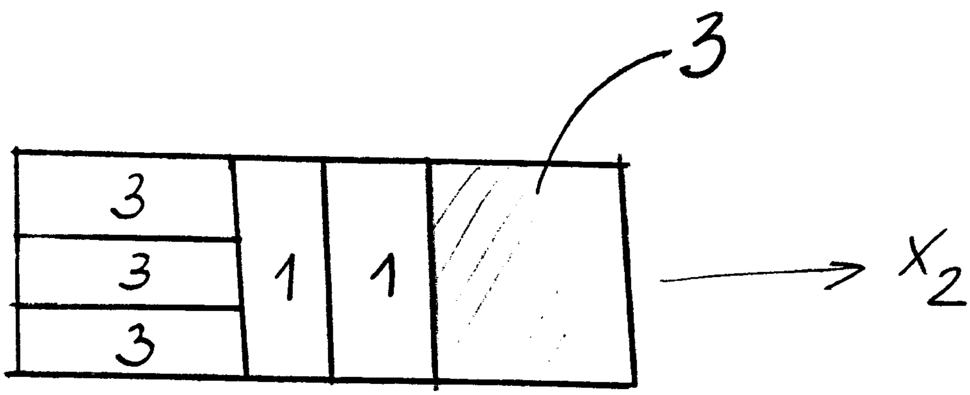
L.V.Kantorovich, 1912–1986 Graduated Leningr. University at 18, full Professor at 22, first paper published at 16, Stalin prize, 1949, Lenin prize, 1965, Nobel prize, 1975.

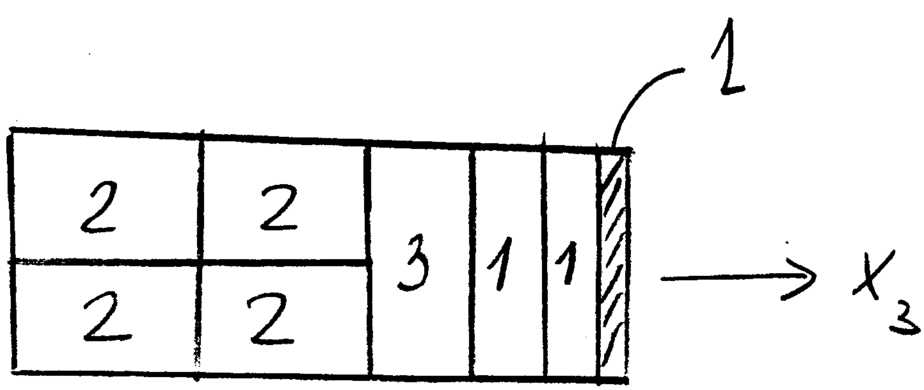
Three breakthroughs in optimization

- Linear programming 1939
- General optimality conditions 1940
- Functional analysis techniques 1944–1948

The Veneer Company Problem
(L. Kantorovich, 1939)







$$z = 2x_1 + 3x_2 + x_3 \to \min$$

$$4x_1 + 2x_2 + 2x_3 \geq 27$$

$$2x_1 + 0 \cdot x_2 + 4x_3 \geq 11$$

$$x_1 + 3x_2 + x_3 \geq 9$$

$$x_1 \geq 0, \;\; x_2 \geq 0, \;\; x_3 \geq 0$$

Diet Problems (Stigler 1945)

	Bread	Meat	Pot.	Cab.	Milk	Gel.	One Day Nec.
Cal.	1254	1457	318	46	309	1725	3000
Pro.	38	73	8	4	16	43	70
Cal.	418	41	42	141	536	-	800
Vit.	-	-	70	860	720	-	500
Price	0.3	1.0	0.05	0.18	0.23	0.48	z

$$z = 0.3x_1 + 1.0x_2 + \ldots + 0.48x_6$$

$$1254x_1 + \ldots + 1725x_6 \geq 3000$$

- -

$$70x_3 + 860x_4 + 720x_5 \geq 500$$

$$x_1 \geq 0, \; \ldots, \; x_6 \geq 0$$

Production Model

Raw Material	OpI	$OpII$	$OpIII$	
x_1	→ $\frac{5m}{unit}$ →	$\frac{3m}{unit}$ →	$\frac{4m}{unit}$ →	\$3
x_2	→ $\frac{2m}{unit}$ –	– →	$\frac{6m}{unit}$ →	\$5
x_3	→ $\frac{6m}{unit}$ →	$\frac{8m}{unit}$ –	– →	\$8
Time Limits	320	400	200	

Company seeks the determinations of the daily number of units to be produced, to maximize the profit,

$$z = 3x_1 + 5x_2 + 8x_3 \rightarrow \max$$

$$5x_1 + 2x_2 + 6x_3 \leq 320$$

$$3x_1 + 8x_3 \leq 400$$

$$4x_1 + 6x_2 \leq 200$$

$$x_1 \geq 0, \; x_2 \geq 0, \; x_3 \geq 0$$

Assembly Line Balancing

A manufacturing company produces a final product that is assembled from 3 different parts.

The parts are manufactured within the company by 2 different department.

	$Max\ \frac{}{Week}$	Product	$Rate$	$\frac{unit}{hour}$
$Dept.$	$Cap.$	$PartI$	$PartII$	$PartIII$
1	100 h.	8	5	10
2	80 h.	6	12	4

x_{ij} – number of hours assigned by Department i to part j

Part I: $8x_{11} + 6x_{21}$

Part II: $5x_{12} + 12x_{22}$

Part III: $10x_{13} + 4x_{23}$

The number of final assembly unit is:

$$y = \min \{8x_{11} + 6x_{21}, 5x_{12} + 12x_{22}, 10x_{13} + 4x_{23}\}$$

Since a final assembly unit includes 1 unit of each of the three parts the total number of final assembly units must equal the smallest number of units avaible.

$$\max z = y$$

$$8x_{11} + 6x_{21} \geq y$$

$$5x_{12} + 12x_{22} \geq y$$

$$10x_{13} + 4x_{23} \geq y$$

$$x_{11} + x_{12} + x_{13} \leq 100$$

$$x_{21} + x_{22} + x_{23} \leq 80$$

$$x_{ij} \geq 0$$

LP Model for Resource Allocation

From the Economic stand point LP seeks the best allocation of limited resources to specific economic activities.

We have n activities with unknown level

m resources, whose maximum availability are given by $b_1, \ldots, b_m$

Each unit of activity j consumes an amount a_{ij} of resources i and produces of profit c_j

Transportation Problem

The transportation model seek the determination of a transportation plan of a single commodity from a number of sources to the number of destinations.

An auto company has three plants:
Los Angelos, Detroit and New Orleans

Major Distribution Centers:
Denver and Miami

The capacities of the plants:
1000, 1500, and 1200 cars.

The quarterly demands at the distinations:
2300 and 1400 cars.

	Denver	Miami	
Los Angelos	\$80	\$215	1000
Detroit	\$100	\$108	1500
New Orleans	\$102	\$68	1200
	2300	1400	

x_{ij} - number of cars transported from the source i to the destination j.

$$z = 80x_{11} + 215x_{12} + 100x_{21} + 108x_{22} + 102x_{31} + 68x_{32}$$

$$x_{11} + x_{12} = 1000$$

$$x_{21} + x_{22} = 1500$$

$$x_{31} + x_{32} = 1200$$

$$x_{11} + x_{21} + x_{31} = 2300$$

$$x_{12} + x_{22} + x_{32} = 1400$$

$$x_{ij} \geq 0$$

Assignment Problem

$$\begin{array}{cccc} & & Jobs & \\ & 1 & 5 & 7 \\ Machines & 9 & 12 & 4 \\ & 10 & 3 & 7 \end{array}$$

$$x_{ij} = \begin{Bmatrix} 1, \text{if mach i} \longrightarrow \text{assign to job j} \\ 0, \qquad\qquad \text{if not} \end{Bmatrix}$$

$$x_{11} + x_{12} + x_{13} = 1 \quad x_{11} + x_{21} + x_{31} = 1$$
$$x_{21} + x_{22} + x_{23} = 1 \quad x_{12} + x_{22} + x_{32} = 1$$

$$x_{31} + x_{32} + x_{33} = 1 \quad x_{13} + x_{23} + x_{33} = 1$$

$$x_{ij} \geq 0, \;\; i = 1, ..., 3, \;\; j = 1, ..., 3$$

$$z = x_{11} + 5x_{12} + 7x_{13} + 9x_{21} +$$
$$+ 12x_{22} + 4x_{23} + 10x_{31} + 3x_{32} + 7x_{33}$$

$$x^*_{ij} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

A Work Scheduling Problem

Day	Number of Full Time Emp. Req.	Cost
1. *Monday*	17	10
2. *Tuesday*	13	9
3. *Wednesday*	15	12
4. *Thursday*	19	15
5. *Friday*	14	14
6. *Saturday*	16	20
7. *Sunday*	11	22

$$x_1, x_2, ..., x_7$$

x_i - number of empl. beginning work on day i

$$\begin{pmatrix}1\\1\\1\\1\\1\\0\\0\end{pmatrix} x_1 + \begin{pmatrix}0\\1\\1\\1\\1\\1\\0\end{pmatrix} x_2 + \begin{pmatrix}0\\0\\1\\1\\1\\1\\1\end{pmatrix} x_3 + \begin{pmatrix}1\\0\\0\\1\\1\\1\\1\end{pmatrix} x_4 +$$

$$\begin{pmatrix}1\\1\\0\\0\\1\\1\\1\end{pmatrix} x_5 + \begin{pmatrix}1\\1\\1\\0\\0\\1\\1\end{pmatrix} x_6 + \begin{pmatrix}1\\1\\1\\1\\0\\0\\1\end{pmatrix} x_7 \geq \begin{pmatrix}17\\13\\15\\19\\14\\16\\11\end{pmatrix}$$

$$z = 10x_1 + 9x_2 + 12x_3 + 15x_4 + 14x_5 + 20x_6 + 22x_7 \rightarrow \min$$

Blending Problem

Candy I II

100 sugar
20 nuts
30 chocolate

I - 20% nuts 10% chocolate

II - 10% nuts

I - $5 II - $3

$$5x_1 + 3x_2 \to \max$$

$$x_1 + x_2 \leq 150$$

$$0.2x_1 + 0.1x_2 \leq 20$$

$$0.1x_1 \leq 30$$

$$x_1 \geq 0, \ \ x_2 \geq 0$$

IBM Capacity Available

Products \ Plants	Capacity per Production	Used unit Rate	Capacity Available
1	1	0	4
2	0	2	12
3	3	2	18
Unit Profit	3	5	

$$IBM\ PC - 2 - x_1,\ \ RISC - 6000 - x_2$$

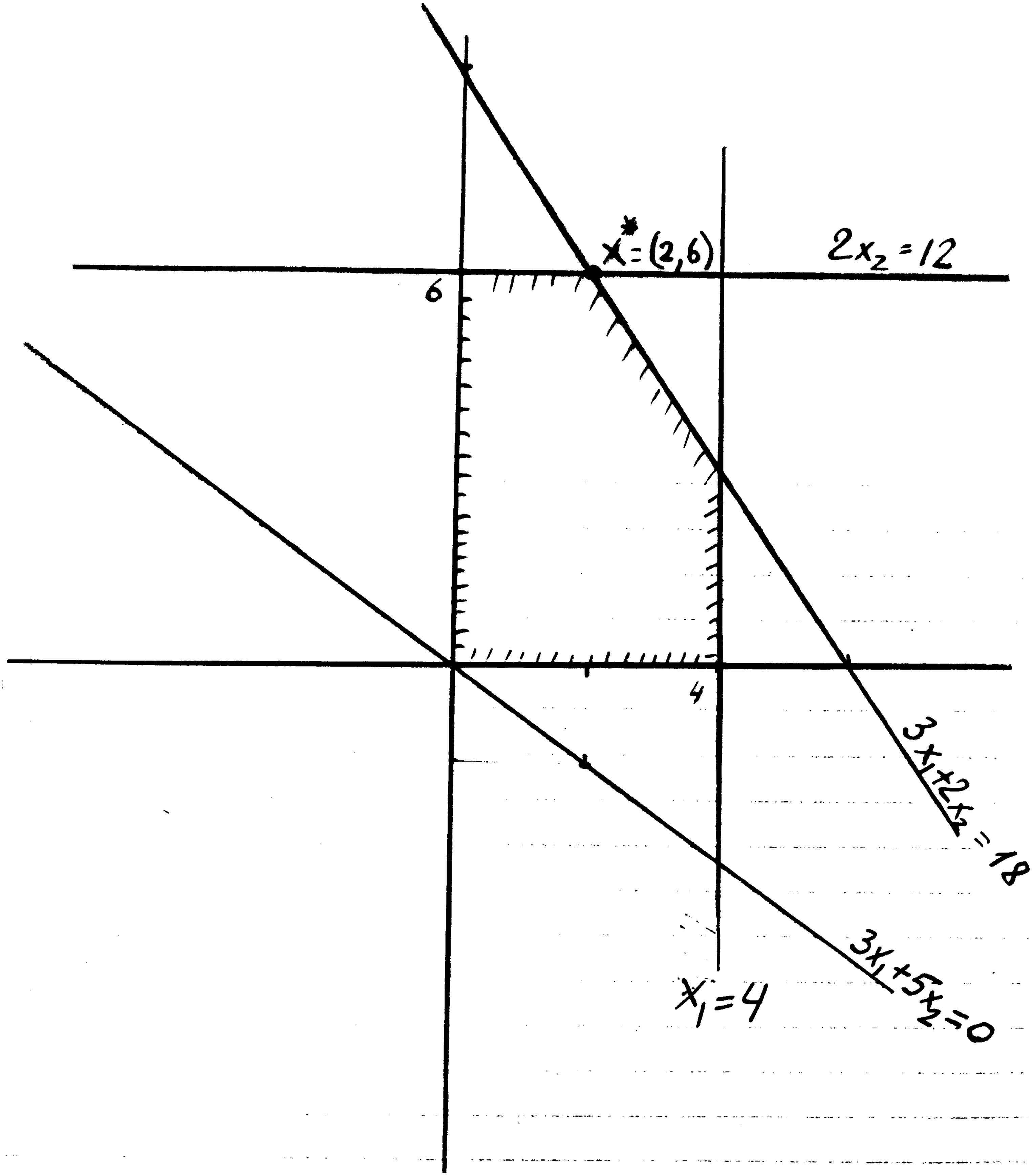
$$£ = 3x_1 + 5x_2 \rightarrow \max$$

$$1 \cdot x_1 + 0 \cdot x_2 \leq 4$$

$$0 \cdot x_1 + 2 \cdot x_2 \leq 12$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1 \geq 0,\ \ x_2 \geq 0$$



$x^* = (2,6)$
$2x_2 = 12$
6
4
$3x_1 + 2x_2 = 18$
$x_1 = 4$
$3x_1 + 5x_2 = 0$

Investment Problem

n projects to invest in

p_i the amount to invest

q_i the profit from project i

$$x_i = \begin{Bmatrix} 1 & \text{if one invests} \\ 0 & \text{if not} \end{Bmatrix}$$

P - the amount of money available

$$z = q_1x_1 + ... + q_nx_n$$

$$p_1x_1 + ... + p_nx_n \leq P$$

$$x_i \in \{0,1\}$$