Applications Of Fat Tailed Models In Financial Markets

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Agenda

- Background & Problem
- Objective & Scope
- Modeling Approach & Verification
- Applications
  - Options Pricing
  - Value at Risk (VaR)
- Conclusion & Recommendation

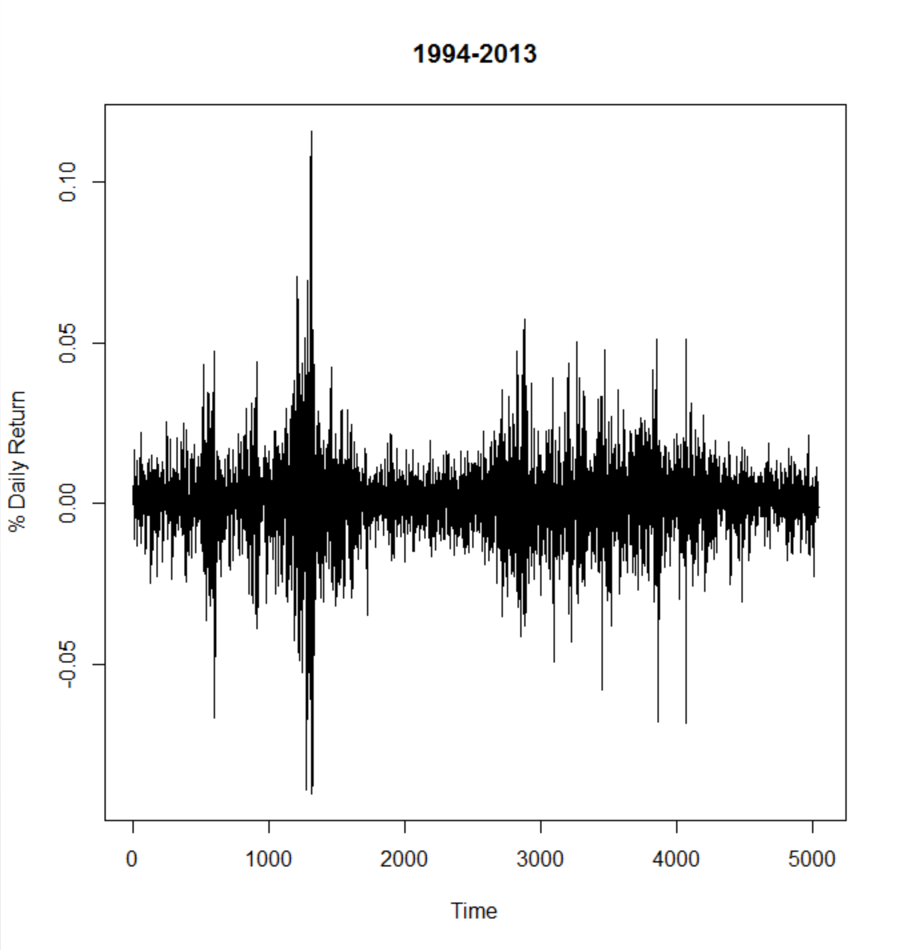
Problem: Fat Tails

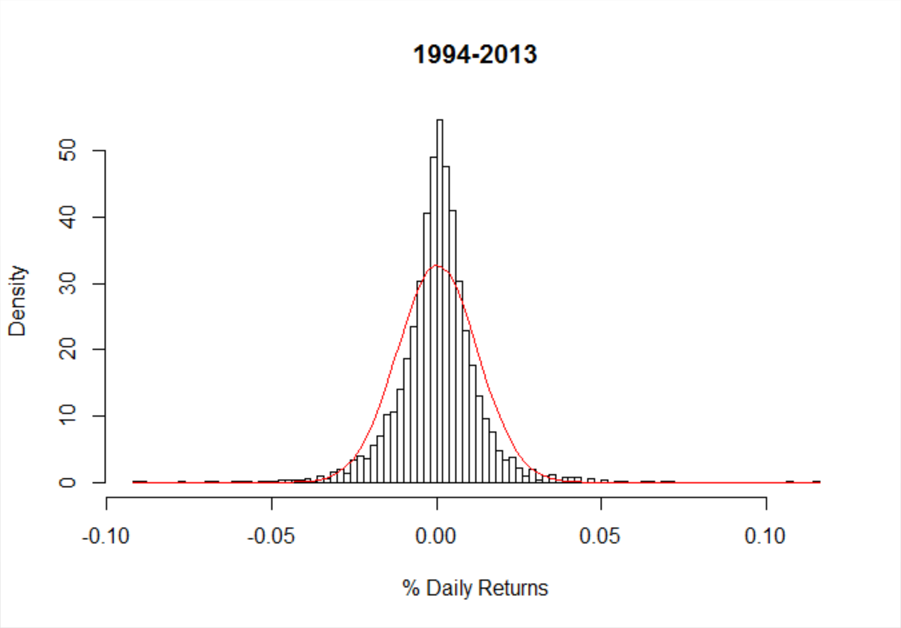
Most financial models are based on normal distribution.

In the real world most data has heavier tails than the normal distribution.

Not taking these heavy tails into account could lead to poor investment decisions.

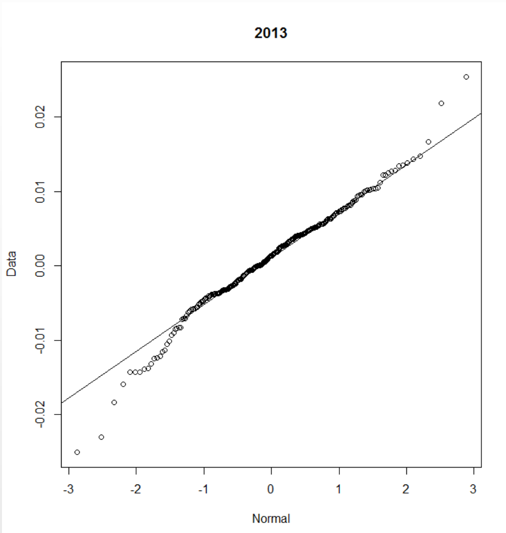
S&P 500 Daily % Returns

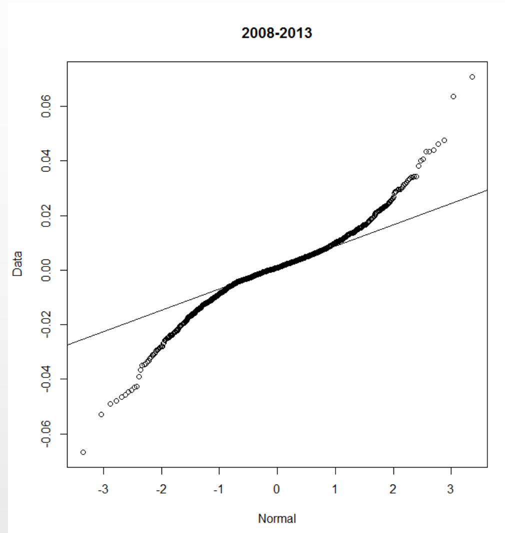


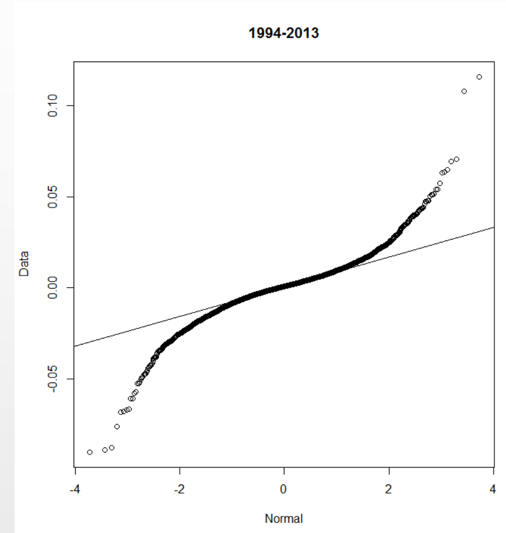


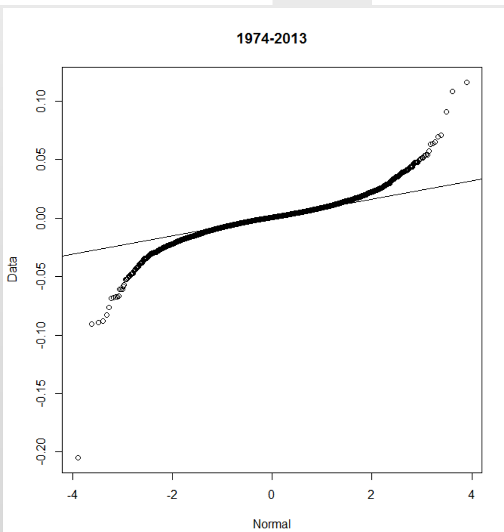
Sample graphs/output shown are for the 20 years dataset only

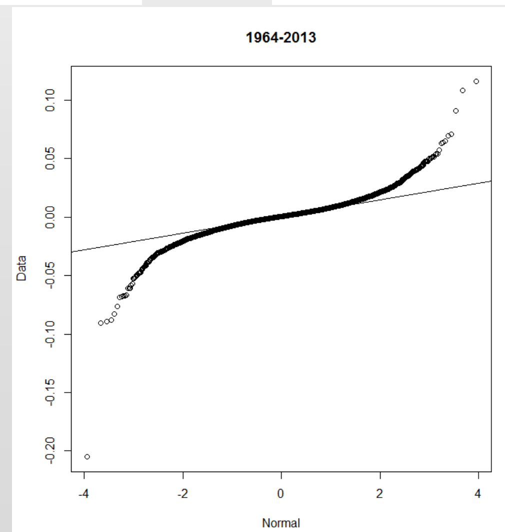
QQ Plots


2013
Data
Normal
-0.02 -0.01 0.00 0.01 0.02
-3 -2 -1 0 1 2 3


2008-2013
Data
Normal
-0.06 -0.04 -0.02 0.00 0.02 0.04 0.06
-3 -2 -1 0 1 2 3


1994-2013
Data
Normal
-0.05 0.00 0.05 0.10
-4 -2 0 2 4


1974-2013
Data
Normal
-0.20 -0.15 -0.10 -0.05 0.00 0.05 0.10
-4 -2 0 2 4

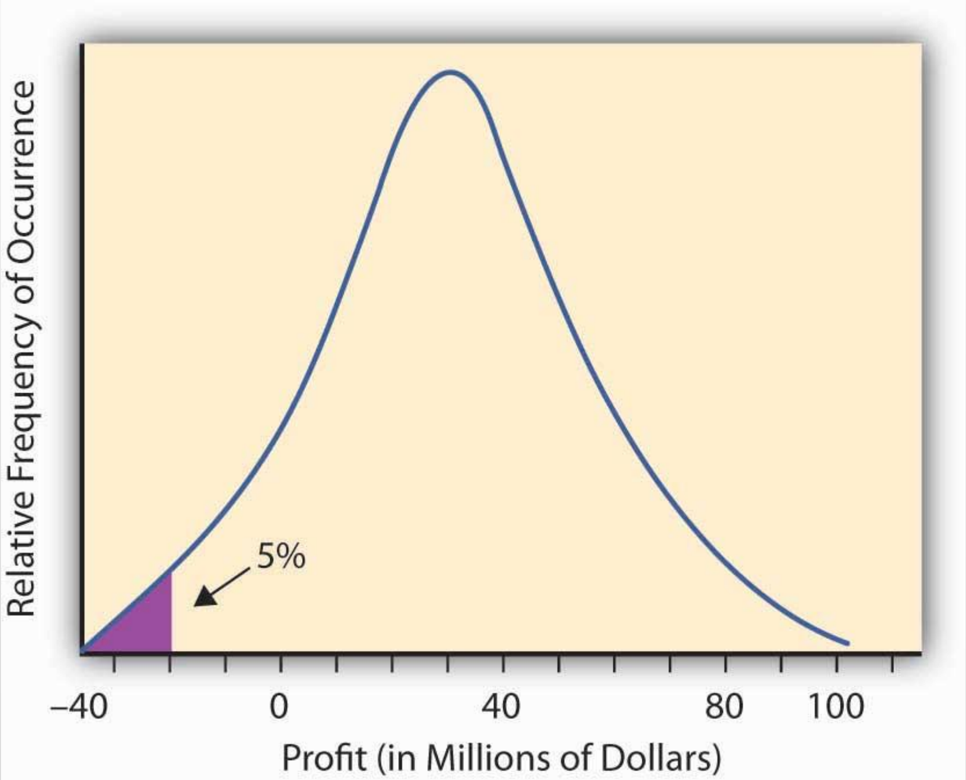

1964-2013
Data
Normal
-0.20 -0.15 -0.10 -0.05 0.00 0.05 0.10
-4 -2 0 2 4

Objective

The aim of this project is to develop a fat-tailed distribution model that fits the S&P500's daily percent returns to be applied in standard financial calculations.

Value at Risk (VaR)

The N-day X% Value at Risk is the maximum loss in N days with a probability of X%



Basel III

"Basel III" is a comprehensive set of reform measures, developed by the Basel Committee on Banking Supervision, to strengthen the regulation, supervision and risk management of the banking sector.

The reform measures utilize VaR, among other metrics, to regulate banks.

Options

- Contract gives buyer the right (but not obligation) to buy/sell underlying asset at predefined "strike" price by a certain date
- Call: Right to buy
  - Value of Max(0, Asset Price – Strike Price)
- Put: Right to sell
  - Value of Max(0, Strike Price – Asset Price)

Most popular pricing method is Black-Scholes which assumes normality and uses volatility (standard deviation)

Scope

- Models will be learned from 1 year, 5 years, 20 years, 40 years and 50 years of daily % change of S&P500
- The final selected model must:
  - Obtain a mean value within 99% CI of the respective data set
  - Obtain a standard deviation value within 99% CI of the respective data set
  - Obtain a kurtosis value within 95% CI of the respective data set
  - Generate a Kolmogorov-Smirnov (K-S) statistic that does not reject that the data arises from the learned model
- Models will then be used to calculate VaR as well as options prices to measure their effectiveness

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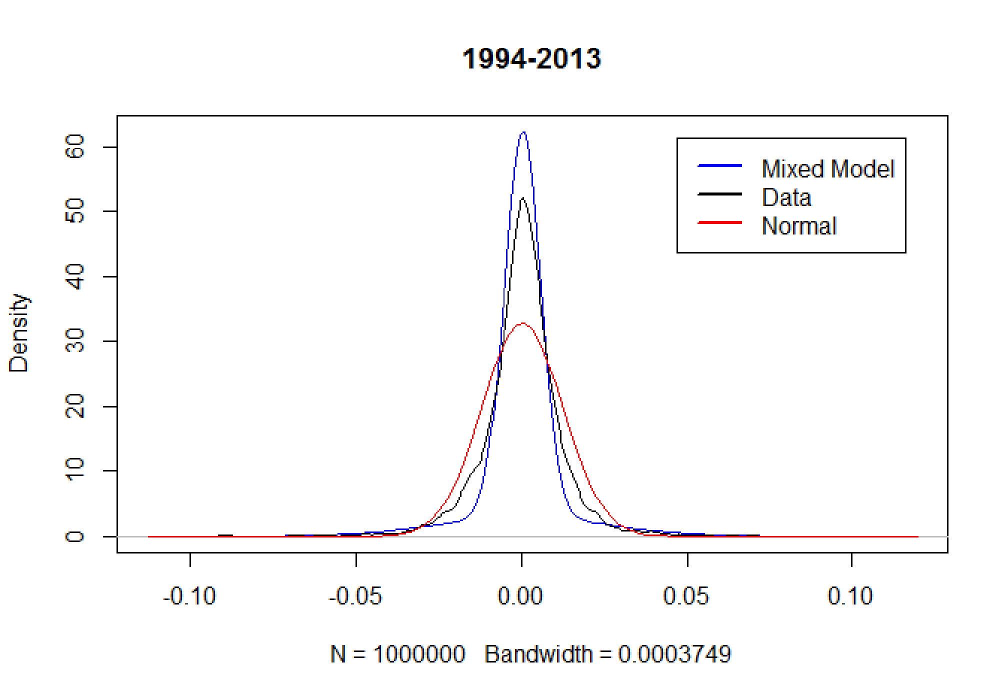
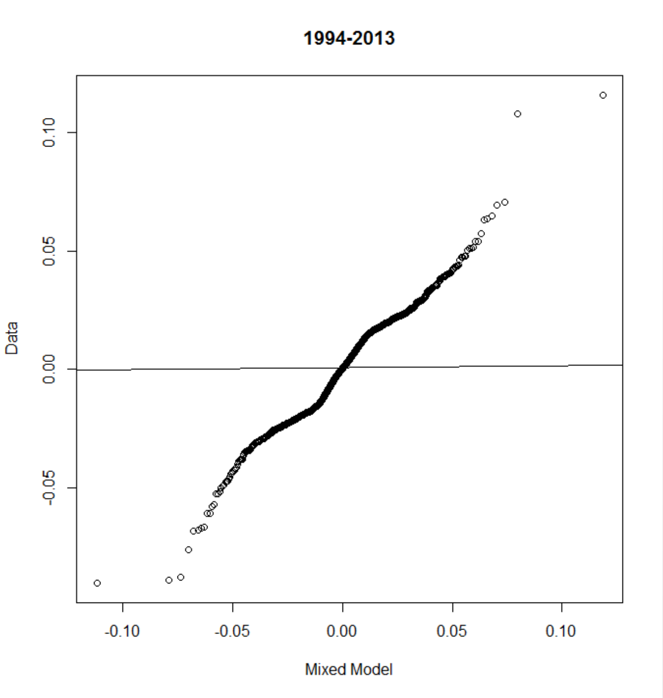
Modeling Approach

- Whole Data
  - Mixed Normal Model (MN)
  - Student's t Distribution (T)
- Extreme Scenarios
  - Generalized Extreme Value distribution (GEV)
  - Generalized Pareto Distribution (GPD)
- Major Assumption:
  - Independent daily % returns

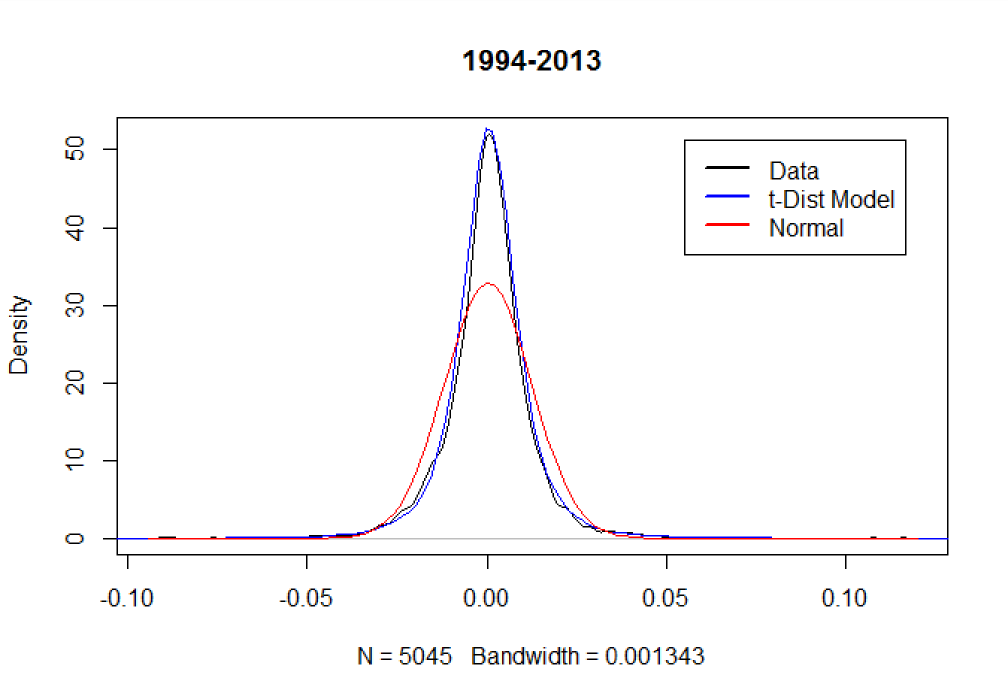
Mixed Normal

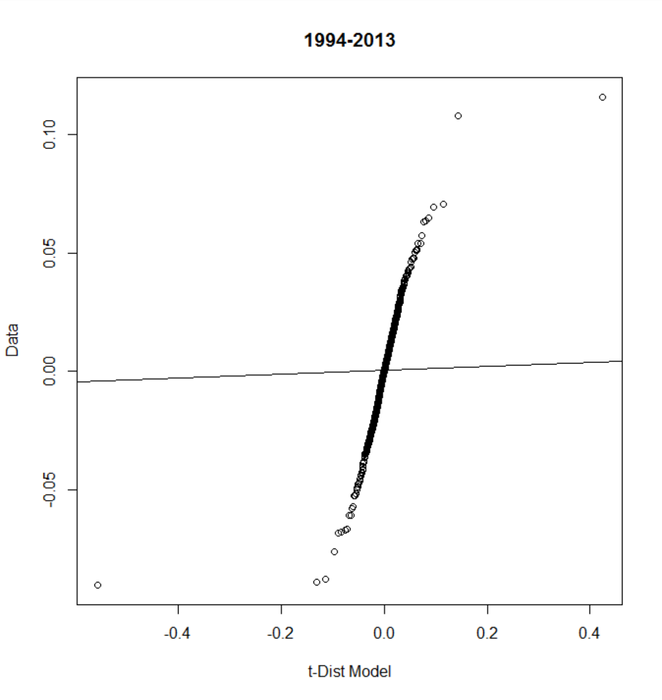




Sample graphs/output shown are for the 20 years dataset only

T-Distribution





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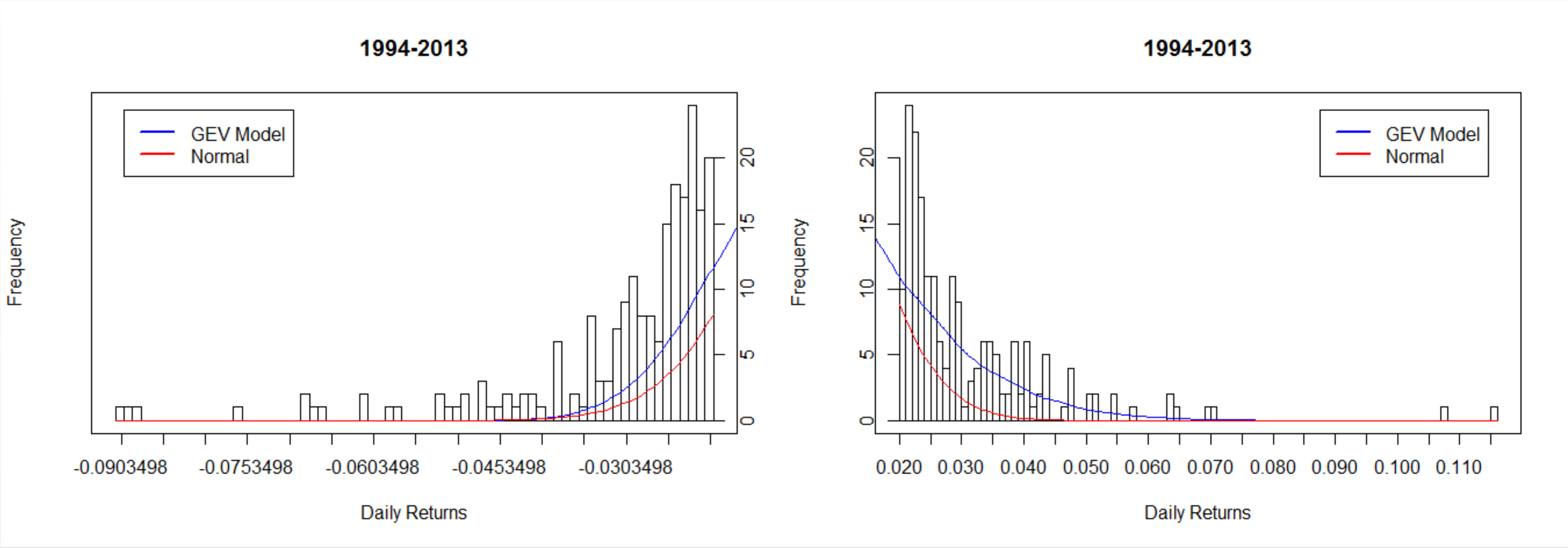
Model Verification

- 99% CI Mean
  - 1 & 5 years' Mixed Normal
- 99% CI Standard deviation
  - All Mixed Normal, 1 & 50 years' T
- 95% CI Kurtosis
  - All Mixed Normal
- K-S Statistic
  - Requirement removed in agreement with the sponsor.

GEV



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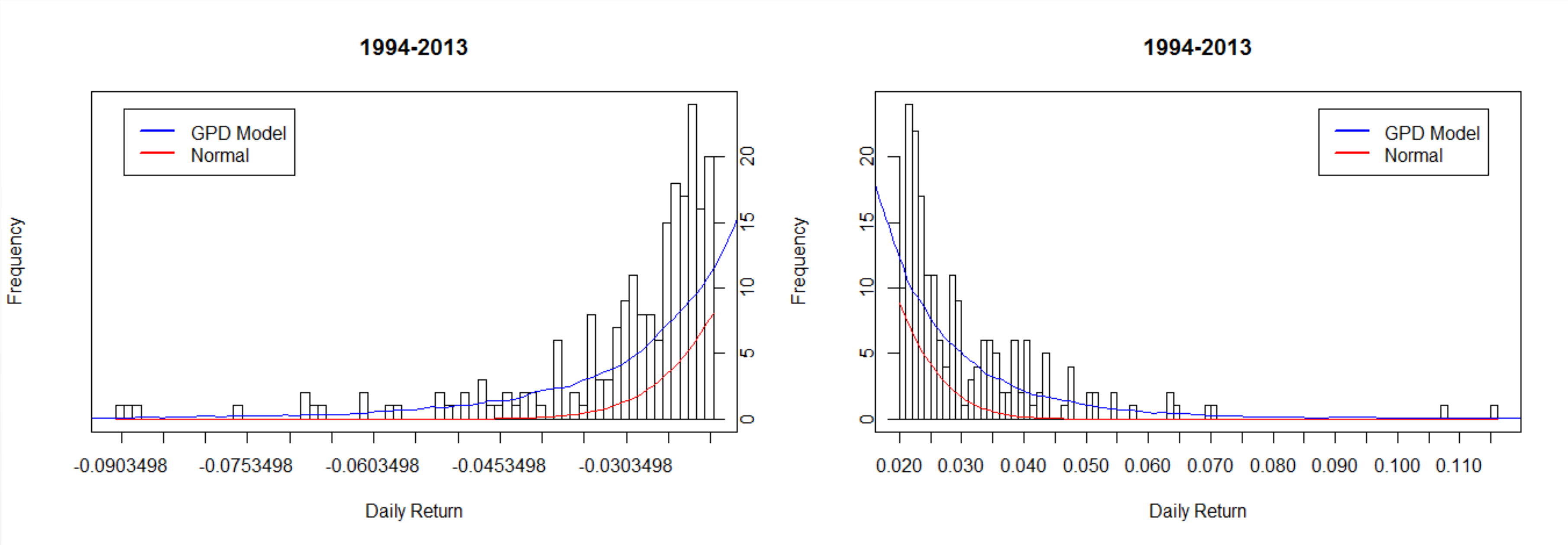
GPD

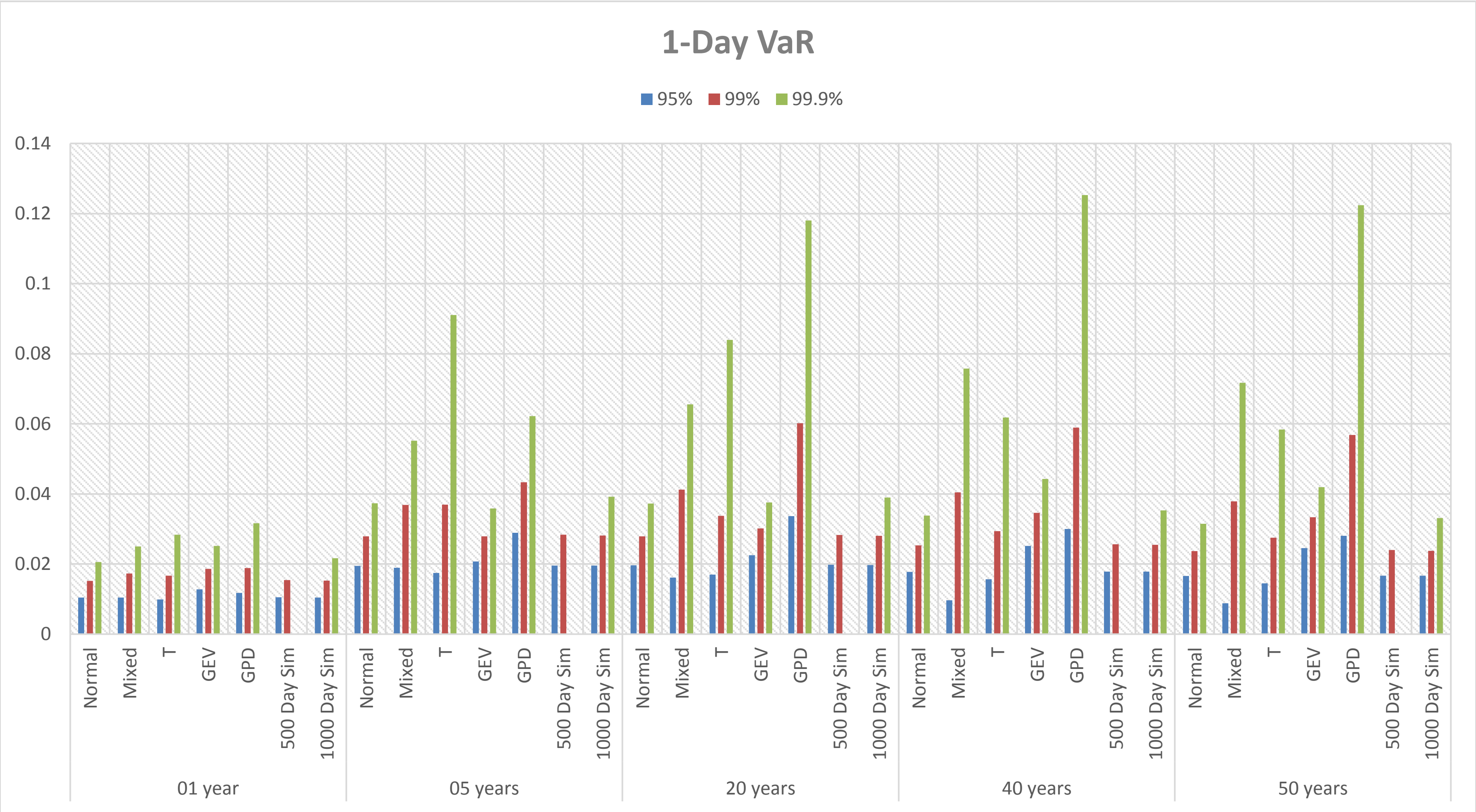


Sample graphs/output shown are for the 20 years dataset only

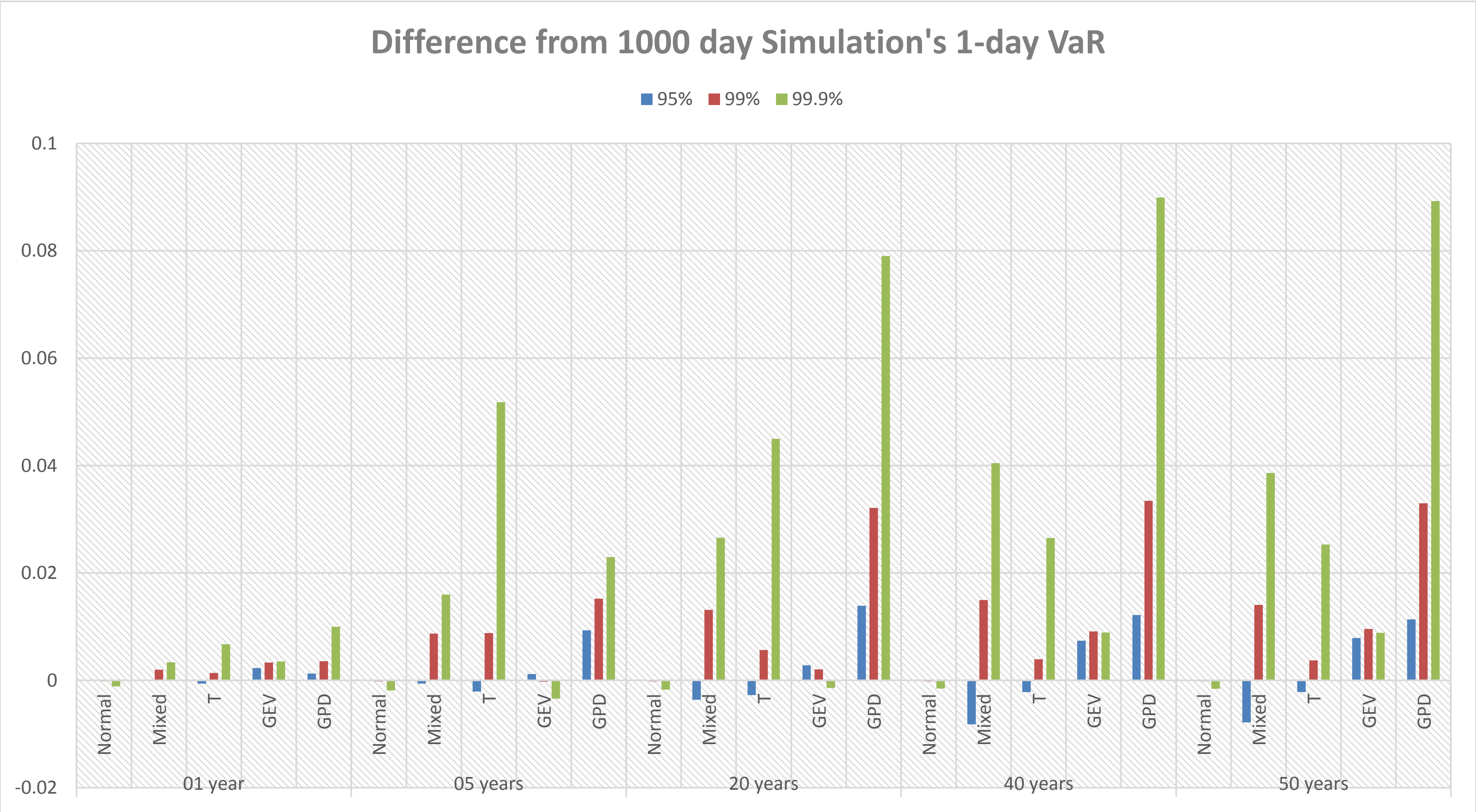
Applications

- Value at Risk (VaR)
  - Calculate 1-day 99.9%, 99% & 95% VaR for all models
  - Compare with 500 and 1000 day Monte Carlo simulation (1,000,000 replications)
- Options Pricing
  - Calculating the price of call and put options for
  - Expirations of
    - 30 days, 60 days, 90 day
  - Strike price of
    - ± 1 %, ± 2 %, ± 5 % changes from start price

Value at Risk



Value at Risk



Options Pricing

Options prices are calculated by

- Simulating the underlying asset (S&P500)'s price for the duration of the option using historical data and the fitted models in a Monte Carlo simulation (assuming a 10% annual drift rate)
- Getting the difference in price from the strike price
- Discounting value taking into account the interest rate
- Repeating for 100,000 simulation runs and then computing the average

We then compare the values generated by the simulation with values calculated by Black Scholes using VIX (implied volatility)

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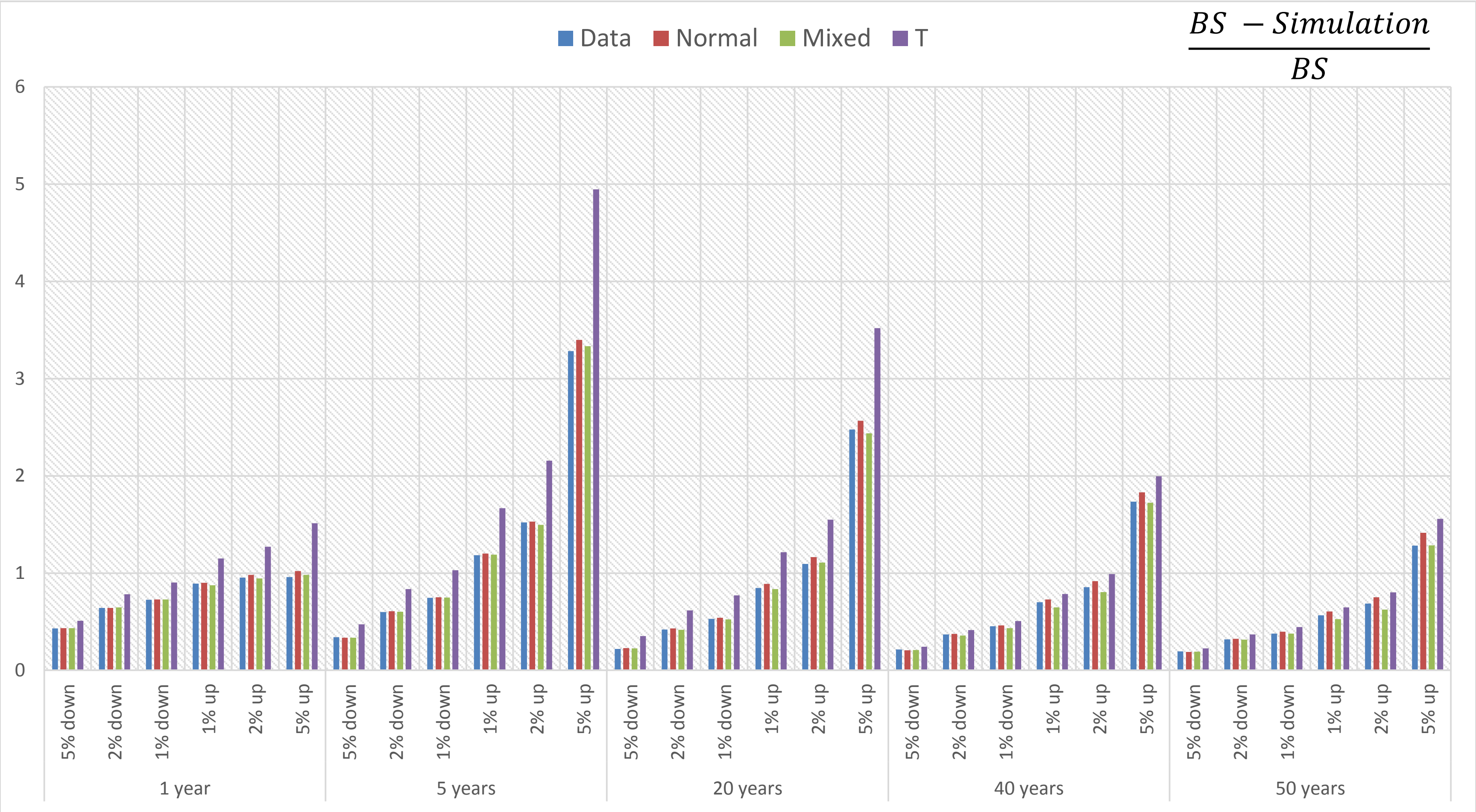
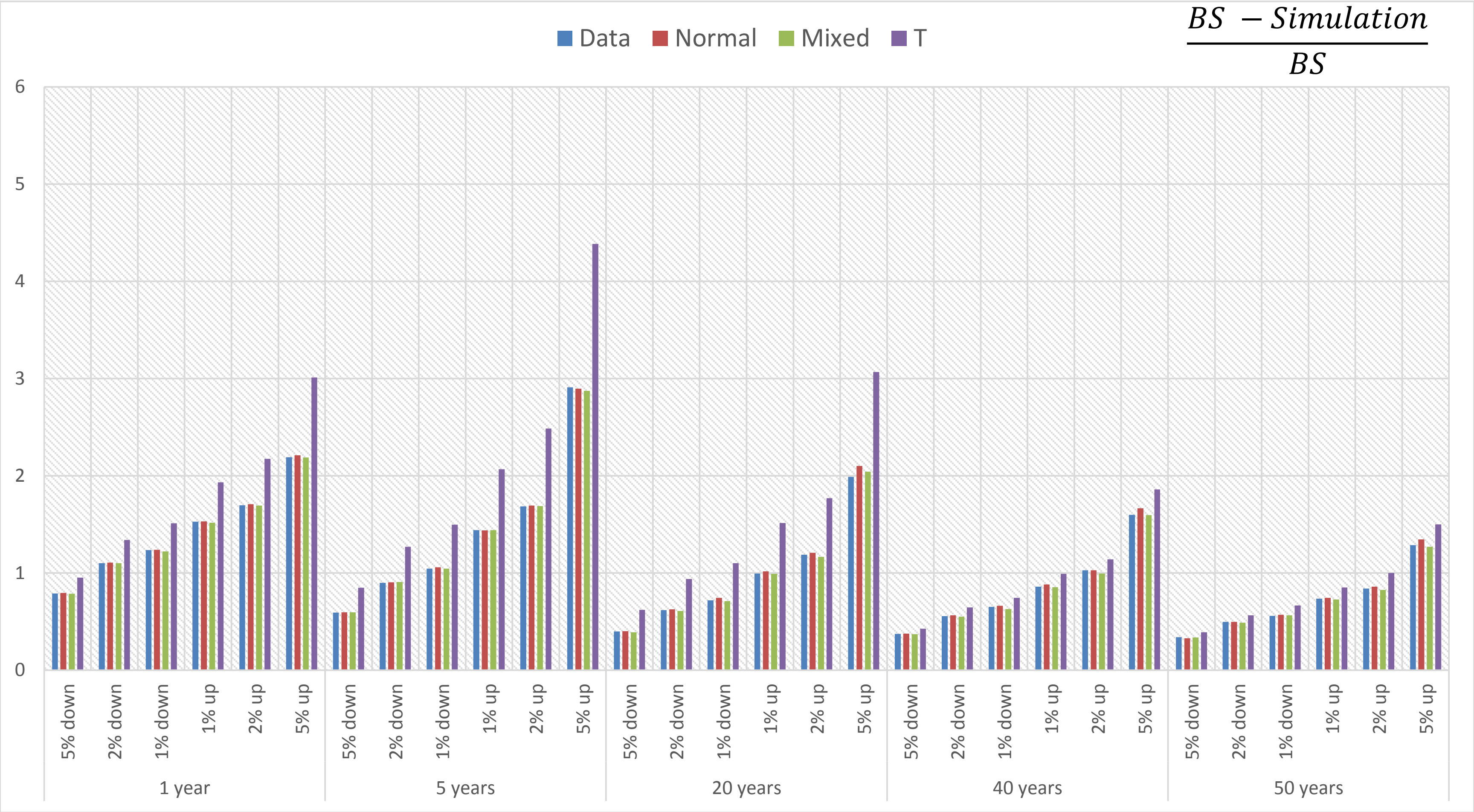
Call Options % Price Differences from Black Scholes (30 days)

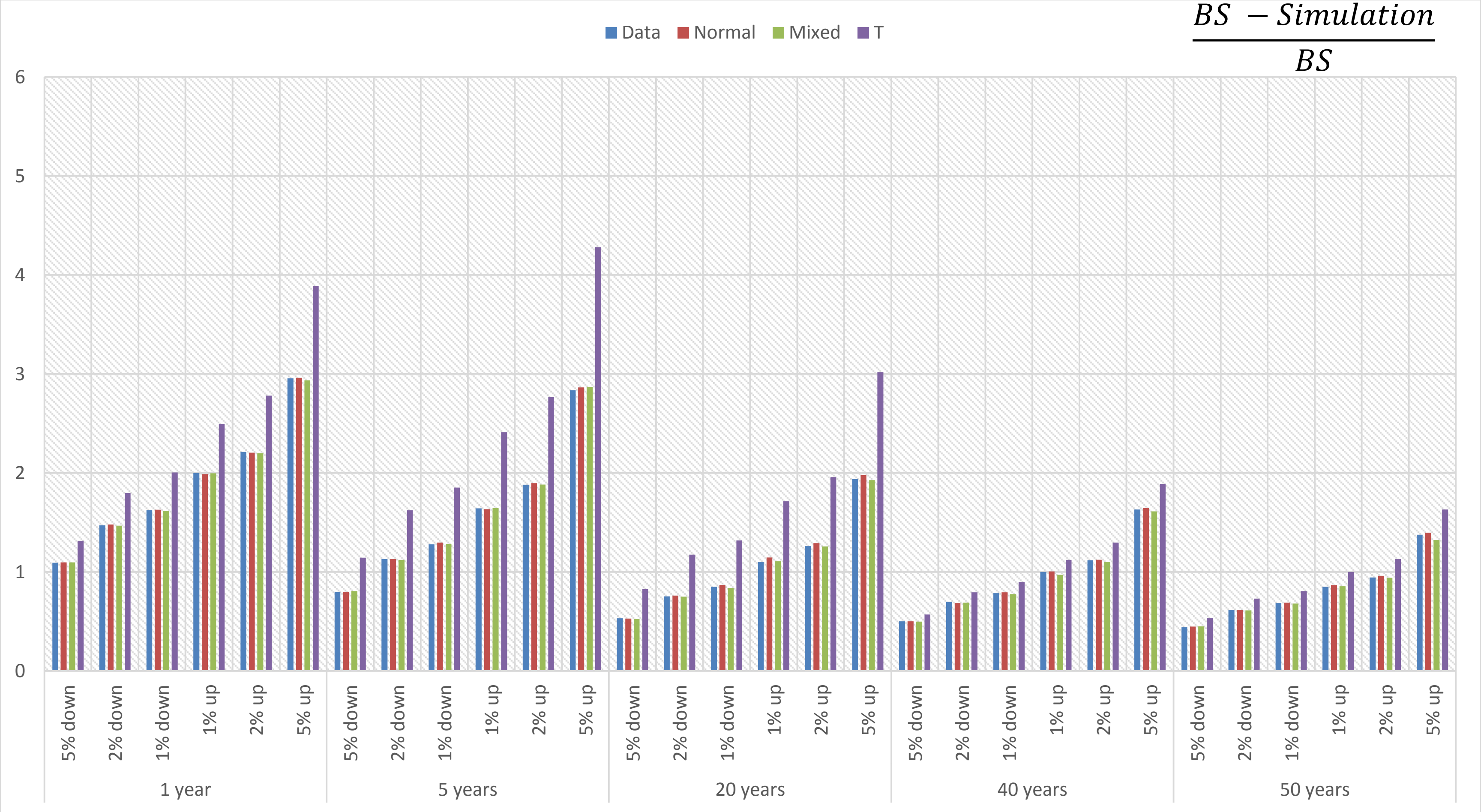


Call Options % Price Differences from Black Scholes (60 days)



Call Options % Price Differences from Black Scholes (90 days)

$$\frac{BS - Simulation}{BS}$$



Summary

Normal models provide the closest results to both, the VaR and options pricing benchmarks

Mixed normal models are generally closer to the benchmarks than the t-distribution models

The **normal** model shows the closest results in magnitude to the simulated VaR and Option prices but is **lower** than they are.

Conclusion

The data does indeed have fatter tails than the normal distribution but the chosen models actually have a fatter tail than the data itself and by a larger magnitude than the difference from normal.

$$Normal_{tail} < Data_{tail} << MixedModel_{tail}$$

Of the developed models, the mixed model would be the next best approximation of the real world.

Recommendation

- Investors should consider both normal and mixed normal models in making investment decisions.
  - Normal model typically underestimates the actual risk
  - Mixed Normal model typically overestimates the actual risk

Next Steps

- Simulate 10-day VaRs to analyze Basel III's requirements
- Compare VaR with Value to Gain
- Regenerate models for log(daily % price change)
- Regenerate models for 10, 15, 25, 30 and 35 years
- Generate a combined model
  - Find the best threshold level
- Investigate potential dynamic volatility of S&P500 daily returns

Thank You

Dr. Kuo Chu Chang

Dr. Kathryn Laskey

Questions?